K25FY3150 Pages: 2 Reg No:................
Name:................

Third Semester FYUGP Degree (Reg) Examination
NOVEMBER 2025
KU3DSCMAT201 - ALGEBRA

2024 Admission onwards

Time : 2 hours Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. If α, β and γ are the roots of $f(x) = 2x^3 + x^2 - 2x - 1 = 0$, find the value of $\sum \alpha^2$.

2. Define an elementary function. Give an example.

3. If α, β and γ are roots of $8x^3 - 4x^2 + 6x - 1 = 0$, find the equation whose roots are $\alpha + \frac{1}{2}$, $\beta + \frac{1}{2}$ and $\gamma + \frac{1}{2}$.

4. Show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

5. Show that $x^5 - 2x^2 + 7 = 0$ has at least two imaginary roots.

6. Write the negation of each statement:
   (a)If she works, she will earn money.
   (b)He swims if and only if the water is warm.

7. Write the converse and contrapositive of the statement ”If productivity increases, then wages rise”.

8. Define truth set of a propositional function. Find the truth set of the propositional function ”$x + 2 > 7$” on the set of positive integers.

Section B

Answer any 4 questions. Each carry 6 marks.

9. If α, β and γ are roots of $f(x) = x^3 - x - 1 = 0$, find the equation whose roots are $\dfrac{1+\alpha}{1-\alpha}, \dfrac{1+\beta}{1-\beta}, \dfrac{1+\gamma}{1-\gamma}$. Hence write down the value of $\sum \dfrac{1+\alpha}{1-\alpha}$

10. Transform the equation $x^3 - 6x^2 + 4x - 7 = 0$ into an equation lacking the second term.

11. If q, r and s are positive, show that the equation $f(x) = x^4 + qx^2 + rx - s = 0$ has one positive, one negative and two imaginary roots.

12. Prove that there are infinitely many primes.

13. Let $p(x)$ be a propositional function defined in the domain $A = \{1, 2, 3, 4, 5\}$, where $p(x) = x^2 < 20$.
(a) Find the truth set of $p(x)$.
(b) Express the statement " For all $x \in A$, $p(x)$ is true " using quantifier and determine it's truth value.
(c) Express the statement " There exists an $x \in A$ such that $p(x)$ is false " using quantifier and find it's truth value.

14. Let $q(x)$ be a propositional function defined on the set **Z** of integers where $q(x) = x^2 - 4x + 3 = 0$.
(a) Determine the truth set of $q(x)$
(b) Express the statement " There exists an integer x such that $q(x)$ is true " using quantifier and negate it.
(c) Express the statement " For all integers x, $q(x)$ is false " using quantifier and negate it.

Section C

Answer any 2 questions. Each carry 14 marks.

15. (a) Find all partitions of $S = \{a, b, c, d\}$.
    (b) Let $\{A_1, A_2, \ldots, A_m\}$ and $\{B_1, B_2, \ldots, B_n\}$ be partitions of a set X. Show that the collection of sets $P = \{A_i \cap B_j\} \setminus \phi$ is also a partition of X.

16. Define partition of a set. Let R be an equivalence relation on a set S. Prove that the quotient set S/R is a partition of S.

17. If α, β, and γ are the roots of $x^4 + 8x^3 + x^2 - x - 10 = 0$, find the values of $\sum \alpha^4$.